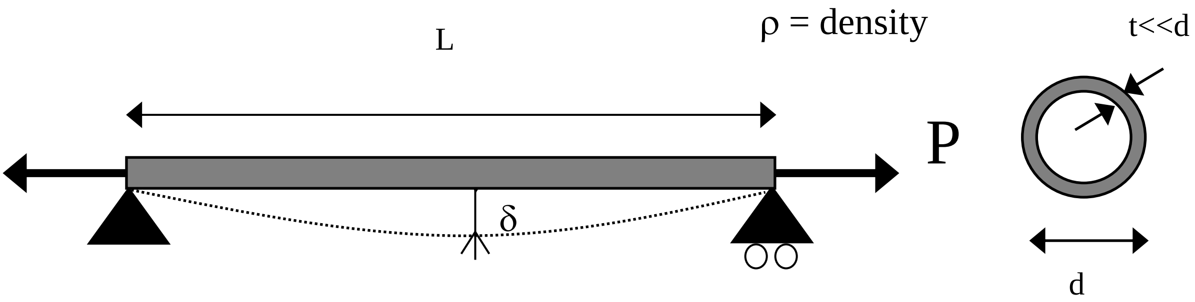
Exercise 5.2.1: Optimization, Search & Exploration 1

In this exercise set, you will see:

- How to formulate and analyze an objective function.
- The study of optimal solutions to an objective function subjected to certain constraints.

Exercise: Optimization of a tube subjected to constraints

The tube below is subjected to an axial force, it's self-weight and constraints relative to its dimensions.



The weight of the tube can be found using the relation defined by the Objective Function (OF):

$$w = \pi \cdot \rho \cdot d \cdot t \cdot L$$

with L = span [m], t = thickness [m], d = diameter [m] et ρ = density [kN/m^3].

The tube must be sized according to the following relations and constraints:

Relations	**Inequality Constraints**
$\delta = \frac{5qL^4}{384EI}$ (deflection due to weight of tube) $q = \pi \cdot \rho \cdot d \cdot t$ (linear load due to weight of tube) $I = \frac{\pi \cdot d^3 \cdot t}{8}$ (moment of inertia)	$\delta \leq 0.001 \cdot L$ (maximum allowable deflection) $t \leq 0.1 \cdot d$ (the validity of the formulas defining I, σ and q)
$\sigma = \frac{P}{\pi \cdot d \cdot t}$	$\sigma \leq \sigma_y$ (strength constraint: stress below yield stress)
	$t \geq 0.005$ (minimum fabrication thickness)

Numerical values:

L [m]	**P** [kN]	ρ [kN/m^3]	σ_y [kN/m^2]	**E** [kN/m^2]
8.0	1500	78.5	150’000	210 E6

Questions:

Without using the numerical values:

a. Transform the constraints (and reduce the equations) that give the relations of types t ≥ f(d) or t ≤ f(d). How many constraints are there and which constraint(s) has/have a relation of the type t ≥ f(1/d) ?

Using the numerical values:

b. Draw the Objective Function on a 2D graph (OF on the y-axis and dt on the x-axis) and the constraints that are in terms of the product *d*t*. What can be said about the minimum of the Objective Function?

c. On a 2D graph (t on the y-axis and d on the x-axis), draw the feasible solution space that satisfies all linear and constant constraints (defined by the function “*d*” or “*t*”). On the same graph, show the minimum Objective Function determined by **b**. What can be said about the solution(s) that minimize(s) the Objective Function? Should a stochastic algorithm be used in this particular case?

d. What constraint(s) is/are never activated for the situation defined by **c**?

e. What can be said about the solution(s) that minimize(s) the objective function with respect to the constraints if the span of the tube is 25m?

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